A Survey on Energy-Efficient Communications

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Abstract—In this paper, we review the literature on physical layer energy-efficient communications. The most relevant and recent works are mainly centered around two frameworks: the pragmatic and the information theoretical approaches. Both of them aim at finding the best transmit and/or receive policies which maximize the number of bits that can be reliably conveyed over the channel per unit of energy consumed. Taking into account both approaches, the analysis starts with the single user SISO (single-input single-output) channel, and is then extended to the MIMO (multiple-input multiple-output) and multi-user scenarios.

I. INTRODUCTION

During the past decade, energy consumption has become an increasingly important issue in wireless networks. For instance, in the current cellular networks, the mobile terminals are equipped with relatively large screens, required to offer more and more functionalities and they need to operate at higher transmission rates for a longer period of time. At the fixed infrastructure level of these networks, the number of base stations has increased dramatically implying important energy costs. According to [2], these costs are expected to be multiplied by a factor of six within the decade 2002–2012. However, significant progress has been made in the art of designing wireless transmitters and receivers. This includes antennas and electronic circuits technology, signal processing algorithms, channel coding techniques and network protocols. The arising question is: Will technological progress be fast enough to control and decrease the energy consumption at the terminal and the network infrastructure sides? Answering such a question is a difficult task and only partial answers can be provided. For this purpose, different communication and information theoretical tools will be used. An important tool and one of the technological breakthroughs in communications is the MIMO concept (i.e., systems composed of multiple antenna terminals) [3][4][5]. It is well known that, for a point-to-point communication, using multiple antenna terminals in full diversity mode (i.e., all the transmit antennas are used to send the same information over the channel) allows one to decrease the transmit power while ensuring a fixed quality of transmission (e.g., the bit error rate).

In this paper, we overview the literature on energy-efficient communications w.r.t. the number of bits that can be reliably conveyed over the channel per unit of energy consumed. The research on this topic has been focused on two main approaches: a pragmatic approach based on practical modulations, coding-decoding schemes, electronics and an information theoretical approach. In Tab. I, we have summarized the general assumptions for both approaches. The systems under investigation consist either of single or multiple antenna terminals. The multi-carrier scenario is a special case of MIMO channel which can be solved in closed-form in the pragmatic approach and, thus, will be considered separately. Regarding the channel coherence time, in the pragmatic approach, the quasi-static channel is considered assuming perfect channel state information at the transmitters (CSIT). The transmitter can adjust its power as a function of the channel state. In the second scenario three types of channels are considered: a) the static channel with perfect CSIT; b) the fast fading channel; c) the slow fading channel. For b) and c) only the statistics of the channel are required at the transmitter. In all scenarios, perfect channel state information is needed at the decoder. The main focus of this paper is the energy-efficiency power allocation (PA) problem although different degrees of freedom are also briefly reviewed. In most of the dedicated literature, only the transmit power at the output of the RF circuits (or the transmit power for reliable data) is considered. Even if this assumption may not be realistic, it allows one to characterize the upper bound on the maximum performance that can be achieved in practice. However, we will also review some works that have taken into account the consumed circuitry energy which may have a critical impact on the system energy-efficiency. Furthermore, only the single-user setting is investigated in the information theoretical approach, whereas for the pragmatic approach the multi-user scenario is also considered.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SYSTEM MODEL AND ASSUMPTIONS FOR THE TWO ENERGY-EFFICIENT APPROACHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionality</td>
<td>Pragmatic approach</td>
</tr>
<tr>
<td>SISO</td>
<td>SISO</td>
</tr>
<tr>
<td>Multi-carrier</td>
<td>Multi-carrier</td>
</tr>
<tr>
<td>MIMO</td>
<td>MIMO</td>
</tr>
<tr>
<td>Number of users</td>
<td>Single-user</td>
</tr>
<tr>
<td>Multi-user</td>
<td>Multi-user</td>
</tr>
<tr>
<td>Coherence time</td>
<td>Quasi-static, CSIT</td>
</tr>
<tr>
<td></td>
<td>Fast fading, CDIT</td>
</tr>
<tr>
<td>Consumed power</td>
<td>RF signal power</td>
</tr>
</tbody>
</table>

A. Notations

We define hereafter some general notations and acronyms that will be used throughout the paper. Let \( R \) denote the
transmit rate, $\gamma$ the received SNR for the single user case or SINR for the multi-user case, $p \in (0, T]$ denote the transmit power which is constrained by $T$, $h$ the channel gain, $\sigma^2$ the noise variance (the noise is assumed Gaussian).

For the MIMO system we denote by $n_t$, $n_r$ the number of available antennas at the transmitter and receiver, $H$ the $n_r \times n_t$ channel matrix, $h_j$ the $j$-th column of $H$, the input covariance matrix is $Q = U \text{diag}(p_1, \ldots, p_{n_t}) U^H$ where $U$ is a unitary matrix, and $p = (p_1, \ldots, p_{n_t})$ is the vector of the corresponding eigenvalues. The average power constraint is $\text{Tr}(Q) = \sum_{j=1}^{n_t} p_j \leq T$. The noise correlation matrix is $\Sigma_z = \sigma^2 I$, unless otherwise specified.

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>SISO</td>
<td>single-input single-output</td>
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<td>MIMO</td>
<td>multiple-input multiple-output</td>
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<tr>
<td>CSIT</td>
<td>channel state information at the transmitter</td>
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<tr>
<td>CDIT</td>
<td>channel distribution information at the transmitter</td>
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<tr>
<td>PA</td>
<td>power allocation</td>
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<tr>
<td>RF</td>
<td>radio frequency</td>
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<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
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<tr>
<td>SINR</td>
<td>signal-to-interference plus noise ratio</td>
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<tr>
<td>CDMA</td>
<td>code division multiple access</td>
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<tr>
<td>BER</td>
<td>bit error rate</td>
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<tr>
<td>FSK</td>
<td>frequency shift keying</td>
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<td>bpcu</td>
<td>bits per channel use</td>
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<td>NE</td>
<td>Nash equilibrium</td>
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<td>OFDMA</td>
<td>orthogonal frequency-division multiple access</td>
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<td>STBC</td>
<td>space-time block coding</td>
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<tr>
<td>AWGN</td>
<td>additive white Gaussian noise</td>
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<td>UPA</td>
<td>uniform power allocation</td>
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</tbody>
</table>

B. A Generic Efficiency Function

The efficiency of a system can be defined in general as the ratio between what the system delivers to what it consumes. For example, we can define the efficiency function as:

$$E(x) = \frac{f(x)}{g(x)},$$

where $x \in [0, \bar{x}]$ denotes the resource constrained by $\bar{x}$, $f(\cdot)$ the benefit function such that $f(0) = 0$ and $g(\cdot)$ is the cost of the resource. We assume also that $g(0) = 0$, which means that the cost in standby mode (no transmission) is zero. The problem of efficient resource allocation is to find the optimal $x^*$ maximizing $E(x)$. Assuming a linear cost, $g(x) = \lambda x$ where $\lambda > 0$ represents the unit cost, then it is sufficient to study the function:

$$\tilde{E}(x) = \frac{f(x)}{x}$$

Depending on the shape of $f(x)$, two types of energy-efficiency functions can be distinguished:

Type I: $f(x)$ is an increasing S-shaped function. In this case, the optimal solution is trivial $x^* \to 0$. For example, for a logarithmic benefit function, $f(x) = a_1 \log(1 + a_2 x)$ with $a_1 > 0$ and $a_2 > 0$ it can be shown that the energy-efficiency function is convex and decreasing w.r.t. $x$. Thus, the optimal solution is trivial $x^* \to 0$. Intuitively speaking, if increasing the resource consumption results in a marginal increase of benefit, then the most efficient solution is not to consume the resource at all.

II. PRAGMATIC APPROACH

We will first study the pragmatic approach, starting with the simplest case of single antenna systems.

A. SISO

In [7][8], the authors study the uplink of a $K$-user CDMA Gaussian channel. A non-cooperative power control game is formulated where the transmitters tune their powers in order to maximize their individual performance in terms of energy-efficiency. The chosen performance metric for the single user case is defined as:

$$G(p, R) = \frac{LR(f(\gamma))}{M p},$$

where $L$ represents the information bits, $M$ the packet size ($M > L$ after the channel coding). Also, $f(\gamma) = (1 - \text{BER})^M$ represents the probability of correct packet reception and BER denotes the bit error rate. In general, $f(\gamma)$ is an S-shaped function. The energy-efficiency is a Type I function and a non-trivial solution $\gamma^* > 0$ exists for the optimization of $\frac{L}{M \gamma R}$. This is illustrated in Fig. 1 for $R = 1$ bpcu (bits per channel use), $M = L = 80$ and a non-coherent FSK modulation [8]. The optimal transmit policy corresponds to the power achieving the optimal SNR $\gamma^*$ while satisfying the power constraint. This result is shown to extend to the multi-user scenario where, at the Nash equilibrium (NE) state (see e.g., [9][10]), the optimal transmit policy for any user is the minimal power that allows it to achieve the optimal SINR equal to $\gamma^*$ (independently of the user identity).

In [11], the authors showed that the performance obtained at the NE is inefficient. In order to obtain a Pareto improvement of the non-cooperative power control game, different methods have been proposed such as: pricing techniques [11], hierarchy among users with either successive interference cancellation.
at the receiver or using the Stackelberg formulation [12] [13], repeated games framework [14].

Several extensions of [8] have been proposed by considering: The influence of other supplementary degrees of freedom on the system energy-efficiency, such as the transmit constellation size [15], transmission rate [16], [20], the coefficients of the receive filter [18], [22], [21]; multi-hop systems and introducing the circuitry consumed power [22], [23]; non-linear receivers [17]. For more details the reader is referred also to [19].

In [24], the non-cooperative power control game is studied in a frequency-selective environment for the uplink of an impulse-radio ultrawideband system. In this case, the problem is more challenging than single path because of the self-interference in addition to multiple access interference and every user achieves a different SINR at the output of its Rake receiver. The authors of [23] study the energy-efficiency non-cooperative power control game in large networks. The nodes are assumed to form clusters and send the local signal to distant receivers. In this scenario, the NE is characterized assuming that the players are the clusters that choose their average transmit power to maximize the energy-efficiency.

B. Multi-carrier

The authors in [25] have extended the analysis in [8] to the study of the PA problem in multi-carrier CDMA systems. The transmitter can send independent data flows over a number of $D \geq 2$ orthogonal carriers. The energy-efficiency utility writes as:

$$G(p, R) = \frac{R_L D}{M} \sum_{d=1}^{D} p_d \gamma_d$$

(5)

where $p = (p_1, \ldots, p_D)$, $p_d \geq 0$, represents the power allocated to the $d$-th carrier and $\gamma_d$ is the receive SNR on the $d$-th carrier. The authors prove that the optimal PA policy is to use only the best carrier (w.r.t. the channel gain) and to transmit over this carrier with a power that achieves an SNR equal to $\gamma^*$. The result is extended to the multi-user case.

A different energy-efficiency function has been studied in [28][29] for multi-carrier frequency-selective OFDMA channels. This function is defined by the ratio of the throughput and the total power (transmit plus circuitry consumption). The throughput is the transmission rate depending on the SNR gap factor.

C. MIMO

The multi-carrier case can be seen as a particular MIMO channel where $n_r = n_t = D$ and $H$ is a diagonal matrix. Now we will focus on the general MIMO case. The major difficulty in extending this pragmatic approach to the general MIMO case is that the output SNR will be strongly related to the encoding-decoding schemes implemented.

In [18], the authors study the SIMO (single-input multiple-output) case where the receiver is equipped with several antennas. The users tune the MMSE receiver coefficients (in this case matrices instead of vectors) and their transmit powers. A large system comparison between the MMSE filter, the matched filter and the decorrelator is also provided. In this case, since the transmitter is equipped with one antenna the problem remains essentially a power control problem. In [27], the framework in [26], is extended to multiuser MIMO wireless systems where each terminal can tune its transmit power, beamforming vector and receiver in order to maximize its own utility. Hence, the transmit covariance matrix is restricted to be a unit rank matrix.

In [30], the authors studied the two extreme cases w.r.t. the tradeoff between the diversity and multiplexing gains brought by MIMO systems: (a) the full multiplexing mode, where the transmitter sends independent data flows over its antennas; (b) the full diversity mode, where the transmitter sends the same information over its antennas.

In case (a), the transmit covariance matrix is diagonal $Q = \text{diag}(p)$ and the efficiency function has the same expression as (5) by replacing $D$ with $n_t$. Here, $\gamma_i$ is the output SNR of the matched filter receiver for the $i$-th component of the transmitted signal: $\gamma_i = p_i h_i^H \left( \Sigma_x + \sum_{j \neq i} p_j h_j h_j^H \right)^{-1} h_i$. The authors proved a similar result as in [25] for the single user case. When independent information is sent over the transmit antennas and assuming a matched filter receiver, the optimal PA policy is beamforming in the direction that requires the minimal power to achieve the target SINR $\gamma^*$:

$$p_i^* = \begin{cases} \min \left\{ \frac{\gamma_i}{\Sigma_x^{-1} h_i^H} \right\}, & \text{if } i = k, \\ 0, & \text{otherwise.} \end{cases}$$

(6)

where $k = \arg\max_{j \in \{1, \ldots, n_t\}} \frac{h_j^H \Sigma_x^{-1} h_j}{\sigma_n^2}$ is the index of the best channel and $\Sigma_x$ is a general positive definite noise covariance matrix. This result was extended to any linear receiver [30].

In case (b), the transmit covariance matrix is a unit rank matrix $Q = vv^H$ where $v_i = \sqrt{p_i}$ for all $i \in \{1, \ldots, n_t\}$. The received SNR at the output of the matched filter (or the MRC receiver) is:

$$\gamma_{\text{MRC}} = \sum_{i=1}^{n_t} \sum_{j=1}^{n_r} \sqrt{p_i} \sqrt{p_j} h_i^H \Sigma_x^{-1} h_j$$

(7)

The energy-efficiency function to be maximized is $G(p, R) = \frac{R_L D}{M} \sum_{d=1}^{D} p_d \gamma_d$ under the power constraint $\sum_{i=1}^{n_t} p_i \leq P$. The problem is more difficult here and a closed-form solution can be obtained only for a particular case where $n_r = n_t = n$, $H = \text{diag}(h_{11}, \ldots, h_{nn})$ and $\Sigma_x = \text{diag}(\sigma_1^2, \ldots, \sigma_n^2)$. Notice that this is the dual case of the one studied in [25] (where the transmitter sends independent information over the parallel sub-channels). The optimal solution corresponds to choosing only the link with the best output SNR (i.e., $k = \arg\max_{j \in \{1, \ldots, n\}} \frac{|h_{ij}|^2}{\sigma_n^2}$) and to transmit with a power that achieves $\gamma^*$. Notice that the same solution was obtained in [25].
There are several works that have studied the energy-efficiency in MIMO channels assuming space-time codes. In [31], the authors evaluate the improvement obtained by using multiple antenna terminals and implementing Alamouti diversity schemes. Assuming a fixed transmission rate and the BPSK input modulation, the MIMO system outperforms the SISO in terms of energy-efficiency if only the transmit power consumption is taken into account. When the circuitry energy consumption is also taken into account, this conclusion is no longer true. However, if the input constellation size can be optimized, the MIMO system can outperform the SISO system, in spite of the higher circuitry energy consumption.

Energy efficiency

In [35][36], the multi-level clustering techniques allowing far-off nodes to communicate to the base station are investigated. Other energy-efficient scheduling mechanisms are reviewed in [33]. In [34], the authors derive an adaptive MIMO approach where the transmitter adapts its modulation and rate and chooses either space-division multiplexing, space-time coding or single-antenna transmission. The authors show that this adaptive technique can improve the energy-efficiency up to 30% compared to non-adaptive systems.

III. INFORMATION THEORETICAL APPROACH

We will now overview the information theoretical approach. One of the first papers addressing energy-efficient communications from this point of view is [37] where the author determines the capacity per unit cost for various versions of the photon counting channel. In [38], the authors studies the discrete memoryless channel where a cost $b[\cdot]$ is assigned to each symbol of the input alphabet. The maximum number of bits that can be transmitted reliably through the channel per unit cost is characterized as follows. Two different scenarios were considered depending on whether the input alphabet, $\mathcal{X}$, contains or not a zero cost symbol: $b[x_0] = 0$ (e.g., the silence conveys information).

Assuming that there is no zero cost symbol, the capacity per unit cost is:

\[
\hat{C} = \sup_{\beta > 0} \frac{C(\beta)}{\beta} = \sup_{\beta > 0} \frac{\sup_{X, \|b[X]\|_\beta \leq \beta} I(X; Y)}{\beta} \tag{8}
\]

where $C(\beta) = \sup_{q(X), \|b[X]\|_\beta \leq \beta} I(X; Y)$ represents the capacity of an input-constrained memoryless stationary channel.

If the input alphabet contains a zero cost symbol, $b[x_0] = 0$, the capacity per unit-cost per unit cost is:

\[
\hat{C} = \sup_{x \in \mathcal{X}\setminus\{x_0\}} \frac{D(q_y | X=x_0) ||q_y | X=x_0)}{b[x]} \tag{9}
\]

Notice that the capacity per unit cost is easier to compute since the optimization is not done over the input probability distributions $q(x)$ but over the symbols of the input alphabet. Furthermore, the divergence between two distributions, $D(\cdot | \cdot)$, is easier to compute than the mutual information, $I(\cdot ; \cdot)$ [43].

In [44], the authors consider the discrete memoryless channel with binary inputs where the 0 is a zero cost symbol. As opposed to [38] where the cost constraint is imposed on each symbol, in [44] the codeword is cost-constrained. In this case, it is not possible to guarantee an asymptotically small error probability and, thus, the Shannon capacity is zero. The capacity per unit energy is defined as the maximum rate, in bits per unit of energy consumed, that can be transmitted over the channel such that the maximum likelihood random coding error exponent is positive. Based on this notion, the authors of [45] define the capacity under a similar finite energy constraint as the maximum total number of bits that can be transmitted with a positive error exponent. Then they analyse the connections between this notion and the capacity per unit energy in [44]. In [46], the authors apply the results in [38], [44] to the wide-sense stationary and uncorrelated scattering (WSSUS) channel.

In the remaining part of the paper, we consider the continuous channels and assume that the input alphabet does not contain zero cost symbols unless otherwise specified.

A. SISO

We start with the static SISO AWGN channel. Following [38], the achievable rate per unit cost is:

\[
\Gamma(p) = \frac{1}{2p} \log_2 \left(1 + \frac{p|h|^2}{\sigma^2}ight) \tag{10}
\]

Notice that $\Gamma(p)$ is a Type II efficiency function. This can also be seen in Fig. 2 where we plot $\Gamma(p)$ for the scenario where $\rho = 10$ dB, $h = 1$. In this case, the capacity per unit cost is achieved when $p^* \to 0$ and given by $\Gamma^* = \frac{1}{2} \log_2 e$. Therefore, in order to be energy-efficient in the sense of the capacity per unit cost, the transmitter has to send information with very low power which implies low data rates. This solution may be realistic in sensor networks but is not acceptable in most common scenarios where minimum communication rates are required. For fast fading channels a similar result is proved in [41].
The case of slow fading channels is considered in [30] [41]. In this case, the Shannon achievable rate is equal to zero. Thus, a different information theoretical energy-efficiency function is proposed:

\[ \Gamma(p, R) = \frac{R[1 - P_{\text{out}}(p, R)]}{p}, \]  

(11)

where \( P_{\text{out}}(p, R) = \Pr \left[ \log_2 \left( 1 + \frac{|h|^2}{\sigma^2} \right) < R \right] \) is the outage probability. The numerator, \( R[1 - P_{\text{out}}(p, R)] \), can be seen as the long-term expected throughput. Assuming Rayleigh fading, the closed-form expression of the outage probability is given by \( P_{\text{out}}(p, R) = 1 - \exp \left\{ -\frac{\sigma^2 R}{p} \right\} \). In this case, \( \Gamma(p, R) \) is a Type I energy-efficiency function and a non-trivial solution exists and is given by: \( p^* = \min \{ \sigma^2 (e^R - 1), T \} \). We observe that this result is very similar to the one obtained in Sec. II where the pragmatic energy-efficiency function is considered. This can be explained by the fact that, as opposed to the static and fast fading cases, slow fading channels, there are outage events (i.e., non-zero error probability) which imply the existence of an non trivial tradeoff between the throughput and power consumption.

A very similar notion with the capacity per unit cost is the minimum energy-per-bit. This notion is defined in [39] for the discrete-time AWGN relay channel. By considering the relay power equal to zero the minimum energy-per-bit becomes:

\[ \varepsilon_b = \lim_{p \to 0} \frac{2p}{\log_2 (1 + \frac{|h|^2}{\sigma^2})} = \frac{2\sigma^2}{|h|^2 \log_2 e}. \]

We observe that the minimum energy-per-bit is the inverse of the capacity per unit cost. In [40], the authors study the AWGN relay channel in the presence of circularly symmetric fast fading. They consider different relaying protocols and provide lower bounds on the minimum energy-per-bit.

B. MIMO

In [41], the authors investigated the case of MIMO channels assuming that the channel matrix \( H \) is a \( n_r \times n_t \) random matrix with i.i.d. standard Gaussian entries. It turns out that for static and fast fading channels, the optimal energy-efficient solution is similar to the SISO case. More precisely, the optimal covariance matrix maximizing the achievable rate per unit cost goes to zero \( \Gamma^* = 0 \). The capacity per unit cost for the static channel is \( \Gamma^* \to \frac{1}{\ln 2} \frac{\Tr(\mathbf{H}^H \mathbf{H})}{\sigma^2} \). The result is extended to fast fading channels.

For the slow fading MIMO channel the problem is much more difficult. In contrast to the static and fast fading cases, the results obtained for the single-antenna case are not necessarily extendable to MIMO channels. In this case, even the optimal solution that minimizes the outage probability is still an open issue. This is due to the fact that the mutual information is a random variable that has an intractable probability distribution, and no closed-form expressions are available for the outage probability. Telatar conjectured in [5] that the optimal transmit policy is to spread all the available power, \( P \), uniformly over a subset of \( \ell \) antennas where \( \ell = \ell(R, \sigma^2) \) is a function of the system parameters. This famous conjecture has been proved for the particular cases: \( n_r = 1, n_t = 2 \) in [47] and \( n_r = 1, n_t \geq 2 \) in [48].

A particular case of interest is the case of UPA transmit policy where \( Q = \frac{1}{n_t} \mathbf{I} \). In [49], the authors conjecture that the energy-efficiency is a quasi-concave function w.r.t. \( p \) and that a non-trivial solution exists \( p^* > 0 \). This is illustrated numerically in Fig. 3 for the scenario: \( n_r = n_t = n \in \{1, 2, 4, 8\} \), \( \rho = 10 \text{ dB}, R = 1 \text{ bpcu} \). We observe that the optimal energy-efficiency value is increasing with the system size and, thus, having several transmit antennas improves the energy-efficiency of the system.

The quasi-concavity property w.r.t. the transmit power \( p \) is important for example in the multi-user scenario. It allows one to prove the existence of NE states for non-cooperative energy-efficient games (see [9]).

In [42], the authors study the tradeoff between the minimum energy-per-bit and the spectral efficiency for wide-band MIMO channels assuming that the input alphabet contains a zero cost symbol and the UPA transmit policy.

IV. CONCLUSIONS

In this paper, we overviewed the literature on energy-efficient communications. The current research is focused on maximizing the number of bits per Joule that can be reliably conveyed through the channel. From an information theoretical point of view, the optimal transmit power allocation policy is trivial for the static and fast fading channels. When slow fading is assumed, a non-trivial solution exists and using multiple-antennas terminals improves the system energy-efficiency. However, these conclusions do not hold necessarily in practical scenarios where the circuitry energy is also considered.

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